Journal of Physics: Conference Series

PAPER • OPEN ACCESS

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To cite this article: I K S Blegur and E Retnowati 2018 *J. Phys.: Conf. Ser.* **1097** 012128

IOP Conf. Series: Journal of Physics: Conf. Series **1097** (2018) 012128 doi:10.1088/1742-6596/1097/1/012128

Designs of goal free problems for learning central and inscribed angles

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Abstract. A mathematics problem solving is usually a non-routine problem that different contexts, concepts or uncommon procedures. There are many strategies to present mathematics problems: with a specific goal or without a specific goal. A problem solving with a specific goal asks students to answer the given question. While a problem without specific goal or goal free does not ask students to solve a specific question, but to solve as many unknown as possibly seen in the problem. According to a Cognitive Load Theory (CLT), mathematics problem in goal free format is more beneficial for students with low prior knowledge since it can reduce extraneous cognitive load. By completing goal free problems, students work forward from "what is known" to any possible move. As a result, students are more likely develop reasoning. Nevertheless, there are very few study of goal free problems in mathematics. This paper discusses how to design goal free problems, particularly for learning angles in circle: central and inscribed angles, which is considered a difficult topic for junior high schoolers. Furthermore, this paper also discusses how to implement this goal free problem in the real classroom.

1. Introduction

Mathematics is a computational learning domain that has a well-structured knowledge building. It consists of operations and algorithms on how to solve problems [1]. Therefore, it has been recommended that problem solving should be the main focus of mathematics learning [2-4]. Problem solving can be viewed as a learning activity to solve complex problem [1, 5] that is a problem that contains new context, concepts or required different procedures, and usually involves reasoning to achieve the solution. According to Schmidt, Loyens, Van Gog & Paas [6] learning through problem solving can facilitate students in providing reasons and abilities to explain the facts of observations.

When students solve problems, the thinking process involves three memory systems in their cognitive structure: sensory memory, working memory, and long term memory [7]. Meanwhile, the technique of presentation of learning materials also determines the effectiveness of learning strategies implemented [8]. Therefore, the presentation of problem solving should lessen the load in the cognitive structure. However, recent research [9] related to the effectiveness of problem solving with goal specified problem strategy suggests that problem solving with such strategy present high cognitive loads for novice students. Consequently the knowledge building process becomes less optimal. Nevertheless, according to Cognitive Load Theory (CLT), mathematical problems presented with a goal free strategy for novice students can reduce cognitive loads [10].

CLT is a theory initially developed by an Australian educational psychologist, John Sweller since the 1980s. According to CLT the learning process needs to pay attention to *cognitive load* processed by working memory and *prior knowledge* stored in long term memory. If the cognitive load is greater than the capacity of working memory then learning will become difficult, and knowledge building will be inhibited if the student's prior knowledge is insufficient [11].

Cognitive load itself can be defined as the amount of information that working memory processes at any one time [10, 12]. CLT believes that there are two types of cognitive loads during learning that affect the working memory: *intrinsic cognitive load* and *extraneous cognitive load* [10, 13]. Intrinsic cognitive load in learning is the cognitive load that is presented by the level of complexity in the given teaching material. The complexity of the material may come from the type of learning material itself and is determined by the prior knowledge possessed by learner. Extraneous cognitive load caused by the instructional procedures like the presentation technique of teaching materials. According to Sweller, et. al. [10], the learning process should minimize extraneous cognitive load. Therefore the technique of presenting the material at the time of learning needs to be considered. Good material presentation will provide a relatively small cognitive load even if the given material has a high complexity.

CLT develops learning strategies based on empirically proven principles. It focuses on finding alternative problem solving strategies apart from conventional strategies such as *means ends analysis* [10-12, 14]. Means ends analysis is a strategy used by problem solvers to reduce the distance to destination by taking the sequence of steps that can be evaluated individually. The use of this strategy involves different interconnected steps such as defining differences between problem states, finding operators to reduce those differences, considering sub goals, and etc. According to Sweller, et. al. means ends strategy is a strategy commonly used by novice students when solving problems presented with goal specified problem. Furthermore, the means ends strategy can solve the problem, but presents a high extraneous cognitive load in working memory [12]. As a result, knowledge development becomes less optimal. Therefore the use of this strategy should be avoided. How to avoid the use of this strategy is to present the problem without specific goal or goal-free [10-12, 14, 15].

Goal free problems are also known as no-goal problems [13]. This is a learning strategy that does not define the ultimate goal in the given problem [10, 12, 15-17]. Goal free problems were the first learning strategy to be investigated in CLT [10]. A goal free problem occurs when conventional problems with specific goals are replaced by problems with non-specific goals (goal free). For example, in a circle learning for Junior High student, the problem would generally require the student to calculate a certain unknown central or inscribed angles on a circle, such as an angle x. In contrast, a goal free problem would not require students to specifically calculate the angle of x, but using more general words such as "determines as many other unknown angles as possible". The more general words used on goal free problems make students free to calculate the other angles they can, which this calculation will still allow the student to calculate the targeted angle of the conventional problem (angle x).

By not giving the specified-goal (goal free), students can use limited working memory capacity to build maximum knowledge [15]. In his research, Ayres [15] concluded that the use of goal free problems can improve problem-solving skills due to the reduced extraneous cognitive load presented in working memory. Ayres constructed a geometry problem where it is only possible to calculate two angle types: the angle to which the goal and the sub-goal angle are. The goal group given (determines the angle value of x) was compared to the goal free group (determining as many unknown angles as possible). Due to the structure of the problem, both groups can only determine the same angle and problem space are identical. However, the results shown by the goal free group make fewer mistakes than the goal given group. According to him, by preventing the use of means ends strategy, students with a goal free approach are able to adopt different problem-solving strategies in a short period of time.

Further research by Ayres [16] on the application of the Pythagorean theorem suggests that a goal free strategy facilitates learning rather than the conventional problem (goal given). Trumpower, Goldsmith, and Guynn [18] found that structurally different transfer problems were solved faster after solving problems with nonspecific goals (goal free) than after solving problems with specific goals (goal given). Wirth, Künsting, and Leutner [19] also found that students who were provided with nonspecific problem solving goals reported lower cognitive load and learned more than students who were provided with specific problem solving goals.

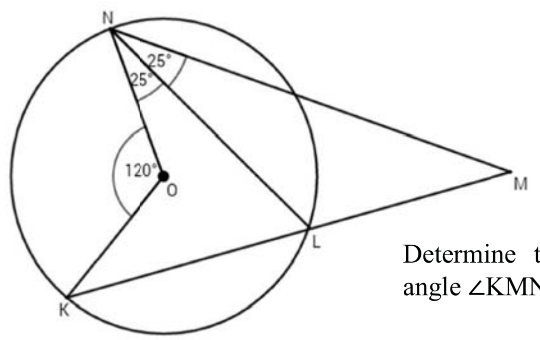
This article aims to propose design of goal free problems for learning angles in circle: central and inscribed angles. Problems solving in this area is mostly difficult to understand since the context is abstract and hardly seen in everyday life. Using the principles of CLT (a literature review), the designs are described. Furthermore, the steps of implement the instruction in the classroom are also discussed.

2. The design of goal free problems for learning central and inscribed angles

Learning of the central or inscribed angles on a circle is one of the materials that must be studied by junior high school student in Indonesia (about 14 years old). In this material the students are facilitated to build knowledge about some definitions and properties of the central and inscribed angles. This material cannot be solved easily because it is abstract and difficult to find its application in everyday life. Thus, the technique of presentation about learning materials is ultimate.

An example of a problem related central or inscribed angles can be presented verbally such as: "Given a rectangle PQRS inscribed in circle with ∠P: ∠Q: ∠R = 5: 8: 7. Determine the measure of angle ∠S! ". Presentation of mathematics problems like this will lead students to make illustrations of existing problems first. After that, using the knowledge about the concept of comparison and the inscribed angles, this problem can be solved. It should be noted that illustrating the problem is also a skill in which the student must have relevant knowledge base. For knowledgeable students, all these stages can be smoothly applied. However, for novice students, a stage may be challenging. Not a few of them will make mistakes at the stage of illustrating the problem before getting to the implementing procedure stage.

On the other hand, problems about the same material can also be presented using a diagram and generally will asks students to calculate the extent of a certain unknown angle, e.g. angle *x*. The presentation of the problem using this strategy is shown by the example in figure 1. Students are directed to determine the measure of the angle ∠KMN, with some known information. Unlike the presentation technique in the first example, the skill of illustrating problem is no needed in here.



Determine the measure of angle ∠KMN.

Figure 1. Example of goal given problem in learning central or inscribed angle.

The problem presentation technique as figure 1 leads students to think about how to determine the measure of angle ∠KMN by using other known angles. In this case, novice students will take a look for angle ∠KMN. To solve the students create a bridgehead with angles' ∠NLM and ∠KLN. When determining the measure of angles' ∠NLM and ∠KLN, students may use the *guess and check* by adjusting between the known information and the alleged truth of the analysis. Although in the end the angle ∠KMN can be measured in size, the students do not think how to apply the relationship theory between the large angles known to the given problem solving structure. As a result students do not get the meaning of the mathematics learning done. Figure 2 illustrates how novice students think backward from the goal to the given information by guessing.

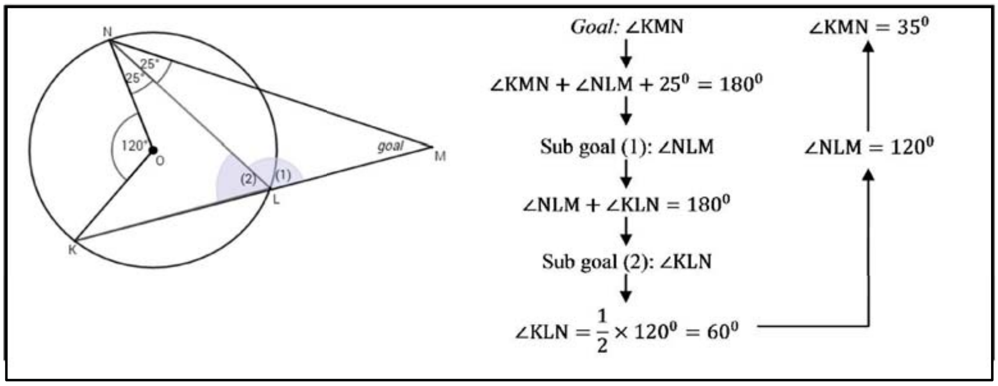


Figure 2. Example of problem solving strategy that use by novice student through goal given problems.

Ayres [16] explains that students make many errors when determining sub-goals and sub-sub-goals before students complete a given goal. This phenomenon is called the *Stage Effect* [12, 15]. According to Ayres [15], the stage effect makes limited working memory capacity used to process inefficient information. As Ayres investigated, stage effect occurs as a result of using means ends strategy. The use of the means ends strategy is indeed a major obstacle to efficient learning [10]. Sweller, Mawer, & Howe [10] described the strategy of means ends analysis does not promoting the rules of induction or acquisition of certain procedural schemes. Although the problem can be solved by using this method, only some parts of the problem solving strategies that can be learned. Hence, students have difficulty in recalling ideas or mathematical ideas that can be used in solving mathematical problems afterwards. Therefore, the use of the means ends strategy should be prevented or minimized.

Preventing or reducing the use of means ends strategies during problem solving can be achieved by removing the specific goals of the problem [16]. Figure 3 provides an example of how to present the problem with this strategy (goal free problem). The problem in figure 3 is a modification of the problem shown in figure 1. In figure 1, the measure of angle ∠KMN is given as the goal of the problem. Meanwhile, in figure 3 the "measure of angle ∠KMN" is replaced with "measure of the unknown angles as much as possible". This statement directs the student to understand the information already given (the angles' ∠ KON, ∠ONL, and ∠LNM) in order to determine the extent of the remaining unknowns as much as possible.

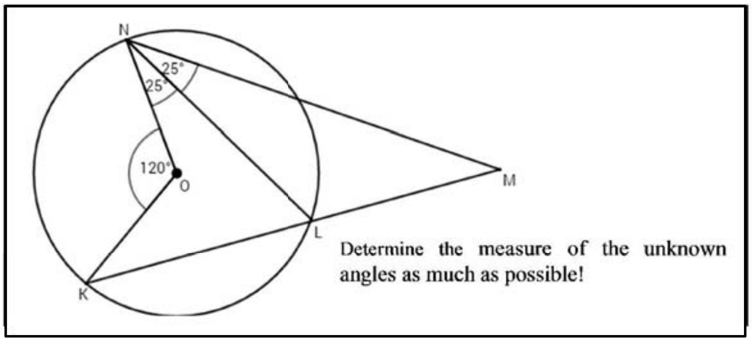


Figure 3. Example of Goal Free Problem in learning central or inscribed angle.

IOP Conf. Series: Journal of Physics: Conf. Series **1097** (2018) 012128 doi:10.1088/1742-6596/1097/1/012128

The required solutions to the problem in figure 3 are the major angle ∠KNO, angle ∠OKL¸ angle ∠KLN, angle ∠NLM, and angle ∠LMN. Instructions "Determine the measure of the unknown angles as much as possible!" makes students look for any angle to calculate its measure. Furthermore this instruction also directs students to be free to determine which angle should be measured first. As a result the student must really understand the information that is known. For example, students want to determine the angle ∠KMN first. This is cannot be done because the known information does not support to find this angle first. As the result, students will move to another angle and think again "which angle can be determined first based on the other known angle". This thought process indicates that students learn to understand "what is known", "Which one can be determined by what is known" and "how to determine it". As described earlier, the goal free problem occurs when the problem is presented without a specific end goal. Figure 4 show the theorems that are used to solve the goal free problems in this case. This is the characteristic of goal free problem for the topic of central and inscribed angle. The problem without a specific end goal leads the student to only have information about what is known and how to solve the problem by using the known information. As the results students will avoid to using the means ends strategy and will solve the problems using the working forward strategy [10]. The principle of working forward strategy will make the development of mathematical knowledge (theorems and algorithm) to be more optimal.

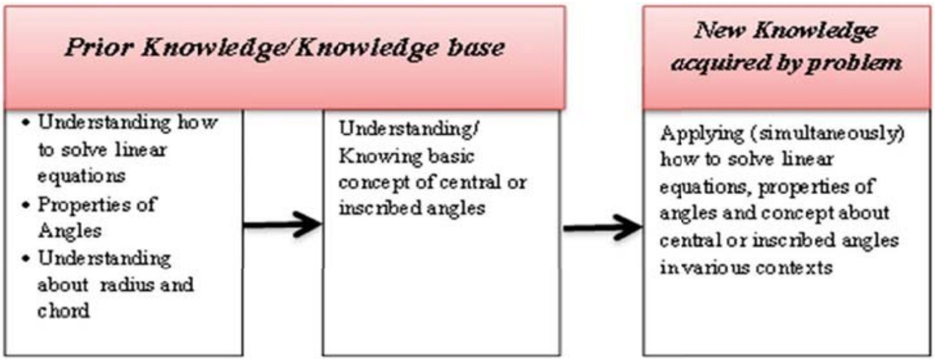


Figure 4. Schema or structured knowledge relevant to the central or inscribed angles problem solving

The thinking process described above is an important process in a meaningful learning. When a specific goal is eliminated, students will learn to understand and how to solve the problem based on the information provided. Sweller, et. al., [10] states that by creating a goal free environment, learning is not dominated by strategies to link the goals with problems. Students are focused only on what is known and how to get to what is being asked. With this working forward principle (without knowing the specific purpose) will make cognitive load on the working memory will be reduced so as to provide more capacity for the process of acquisition and automation of knowledge [17, 20, 21]. In addition to the principle of working forward can reduce the interactivity between elements of the information being processed in working memory. Decreased elemental interactions may reduce the extraneous cognitive load so the learning process can be optimal [10]. This process allows students to construct a meaningful understanding of the material that they learn.

3. Implementation of the Goal free Problems Based Instruction

As mentioned earlier, goal free problems are a technique to present the problem undertaken to reduce extraneous cognitive load in the learning process. Teachers must be sure that students are novices because this instruction is most suitable for novices. Meanwhile this presentation technique can be implemented in the learning process by following the following learning steps in table 1.

Table 1. Learning Steps Using Goal free Problems Strategy.

Activity	Descriptions
Preliminary	1. Apperception: At this stage the teacher provides a stimulus in the form of questions related to the prerequisite material required at the time of learning. It aims to arouse students' memories of these prerequisite materials. 2. Motivation: At this stage the teacher provides information related to the usefulness of the material to be learned in everyday life so that students are eager to follow the learning process. .
Main	a. *Introductory Phase*: the phase in which students are introduced to the theorems that must be understood in the learning materials. Teachers can use learning strategies with guidance such as *Worked Example* [9, 10, 22, 23] to help students construct this knowledge. b. *Acquisition Phase:* a phase in which students learn to solve complex problems using goal free problems strategies. Here are the steps in this phase: 1. Students are given a worksheet as a learning media. This worksheet is designed by following the principle of goal free problems as described previously. At this stage, the teacher can provide an example about how to solve a goal free problem. After that, the students are given the opportunity to solve the problems that exist on the worksheet with a certain time allocation. 2. Students present the worksheet results in a verbal way and then the other students and teachers give their responsibility. 3. Teachers give quiz and discuss the quiz results.
Final	The teacher guides the student to conclude the learning outcomes. The teachers provide information on further learning activities.

In the main activities, teachers can organize learning with individual and collaborative settings. Some experts [15, 16, 19, 21] suggest that learning with goal free strategy is more effectively studied individually than collaboratively. On the other hand, not a few experts claim that collaborative learning is better than individual learning [6, 23-26]. Collaborative learning is a social context-based learning that typically allocates three or more students into small groups where they work together and learn from one another while trying to complete a problem-solving task. According to Schmidt, et. al. [6] collaborative learning in problem solving environments has two functions: activating prior knowledge among group members in group discussions and facilitating students to share expertise. Both of these are important for problem-solving learning. From the cognitive aspect, collaborative learning is well used because by activating prior knowledge can reduce the cognitive load on working memory and by working together in one group, intrinsic cognitive load can also be reduced due to cognitive division among group members[6, 23, 26]. But all these things still need to be proven empirically, so further study on the implementation of collaborative goal free problems strategy is still needed.

4. Conclusion

Problem solving for learning a central and inscribed angles can be presented in goal free problems. This topic possibly performs open solution and has the characteristic for goal free problem. The complexity of the problem requires several steps to perform a final solution which the preceeding steps depends on the solution of the first steps. Learning this topic using goal free problems, students are not asked to solve a particular angle using the central and inscribed angle properties, but instead to solve as many angles as possible. The complexity of central and inscribed angle problem solving is that it involves not only the basic concept about central and inscribed angle it self, but also other knowledge, such as linear equation of one variable, properties of angles and the concept about radius and chords. How to illustrate the problem using circle would also increase the complexity. Therefore, to ensure the goal free problems can be solved and learned effectively by novice students, they should be introduced these basic concepts, and then implement them simultaneously in solving the goal free problems. Furthermore, about the implementation of this strategy, teacher must be sure that the students are novices because this instruction is most suitable for novices.

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